Lineer olmayan denklem sistemleri :

$$\begin{aligned} f_1(x_1, x_2, \ldots, x_n) &= 0, \\ f_2(x_1, x_2, \ldots, x_n) &= 0, \\ \vdots \qquad\qquad & \quad \vdots \\ f_n(x_1, x_2, \ldots, x_n) &= 0, \end{aligned}$$

Örnek :

$$f_1(x_1, x_2, x_3) = 3x_1 - \cos(x_2 x_3) - \frac{1}{2},$$

$$f_2(x_1, x_2, x_3) = x_1^2 - 81(x_2 + 0.1)^2 + \sin x_3 + 1.06,$$

$$f_3(x_1, x_2, x_3) = e^{-x_1 x_2} + 20x_3 + \frac{10\pi - 3}{3},$$

Sabit Nokta iterasyonu :

$$3x_1 - \cos(x_2x_3) - \frac{1}{2} = 0,$$

$$x_1^2 - 81(x_2 + 0.1)^2 + \sin x_3 + 1.06 = 0,$$

$$e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3} = 0.$$

$$x_1 = \frac{1}{3}\cos(x_2x_3) + \frac{1}{6},$$

$$x_2 = \frac{1}{9}\sqrt{x_1^2 + \sin x_3 + 1.06} - 0.1,$$

$$x_3 = -\frac{1}{20}e^{-x_1x_2} - \frac{10\pi - 3}{60}.$$

$$g_1(x_1, x_2, x_3) = \frac{1}{3}\cos(x_2x_3) + \frac{1}{6},$$

$$g_2(x_1, x_2, x_3) = \frac{1}{9}\sqrt{x_1^2 + \sin x_3 + 1.06} - 0.1,$$

$$g_3(x_1, x_2, x_3) = -\frac{1}{20}e^{-x_1x_2} - \frac{10\pi - 3}{60}.$$

$$\mathbf{x}^{(0)} = (0.1, 0.1, -0.1)^t.$$

$$x_1^{(k)} = \frac{1}{3}\cos x_2^{(k-1)} x_3^{(k-1)} + \frac{1}{6},$$

$$x_2^{(k)} = \frac{1}{9}\sqrt{\left(x_1^{(k-1)}\right)^2 + \sin x_3^{(k-1)} + 1.06} - 0.1,$$

$$x_3^{(k)} = -\frac{1}{20}e^{-x_1^{(k-1)}x_2^{(k-1)}} - \frac{10\pi - 3}{60}$$

$$\left\|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\right\|_\infty < 10^{-5}.$$

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$\left\|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\right\|_\infty$
0	0.10000000	0.10000000	−0.10000000	
1	0.49998333	0.00944115	−0.52310127	0.423
2	0.49999593	0.00002557	−0.52336331	9.4×10^{-3}
3	0.50000000	0.00001234	−0.52359814	2.3×10^{-4}
4	0.50000000	0.00000003	−0.52359847	1.2×10^{-5}
5	0.50000000	0.00000002	−0.52359877	3.1×10^{-7}

$$\mathbf{p} = \left(0.5, 0, -\frac{\pi}{6}\right)^t \approx (0.5, 0, -0.5235987757)^t,$$

Düzeltilmiş Sabit Nokta Yöntemi :

$$x_1^{(k)} = \frac{1}{3}\cos\left(x_2^{(k-1)} x_3^{(k-1)}\right) + \frac{1}{6},$$

$$x_2^{(k)} = \frac{1}{9}\sqrt{\left(x_1^{(k)}\right)^2 + \sin x_3^{(k-1)} + 1.06} - 0.1,$$

$$x_3^{(k)} = -\frac{1}{20}e^{-x_1^{(k)} x_2^{(k)}} - \frac{10\pi - 3}{60}.$$

$$\mathbf{x}^{(0)} = (0.1, 0.1, -0.1)^t.$$

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$\left\|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\right\|_\infty$
0	0.10000000	0.10000000	−0.10000000	
1	0.49998333	0.02222979	−0.52304613	0.423
2	0.49997747	0.00002815	−0.52359807	2.2×10^{-2}
3	0.50000000	0.00000004	−0.52359877	2.8×10^{-5}
4	0.50000000	0.00000000	−0.52359877	3.8×10^{-8}

Newton Yöntemi :

$$A(\mathbf{x}) = \begin{bmatrix} a_{11}(\mathbf{x}) & a_{12}(\mathbf{x}) & \cdots & a_{1n}(\mathbf{x}) \\ a_{21}(\mathbf{x}) & a_{22}(\mathbf{x}) & \cdots & a_{2n}(\mathbf{x}) \\ \vdots & \vdots & & \vdots \\ a_{n1}(\mathbf{x}) & a_{n2}(\mathbf{x}) & \cdots & a_{nn}(\mathbf{x}) \end{bmatrix},$$

$$J(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(\mathbf{x}) & \frac{\partial f_1}{\partial x_2}(\mathbf{x}) & \cdots & \frac{\partial f_1}{\partial x_n}(\mathbf{x}) \\ \frac{\partial f_2}{\partial x_1}(\mathbf{x}) & \frac{\partial f_2}{\partial x_2}(\mathbf{x}) & \cdots & \frac{\partial f_2}{\partial x_n}(\mathbf{x}) \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1}(\mathbf{x}) & \frac{\partial f_n}{\partial x_2}(\mathbf{x}) & \cdots & \frac{\partial f_n}{\partial x_n}(\mathbf{x}) \end{bmatrix},$$

$$\mathbf{G}(\mathbf{x}) = \mathbf{x} - J(\mathbf{x})^{-1}\mathbf{F}(\mathbf{x}),$$

$$\mathbf{x}^{(k)} = \mathbf{G}(\mathbf{x}^{(k-1)}) = \mathbf{x}^{(k-1)} - J(\mathbf{x}^{(k-1)})^{-1}\mathbf{F}(\mathbf{x}^{(k-1)}).$$

Örnek :

$$3x_1 - \cos(x_2x_3) - \frac{1}{2} = 0,$$

$$x_1^2 - 81(x_2 + 0.1)^2 + \sin x_3 + 1.06 = 0,$$

$$e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3} = 0$$

$$f_1(x_1, x_2, x_3) = 3x_1 - \cos(x_2x_3) - \frac{1}{2},$$

$$f_2(x_1, x_2, x_3) = x_1^2 - 81(x_2 + 0.1)^2 + \sin x_3 + 1.06,$$

$$f_3(x_1, x_2, x_3) = e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3},$$

$$J(x_1, x_2, x_3) = \begin{bmatrix} 3 & x_3 \sin x_2x_3 & x_2 \sin x_2x_3 \\ 2x_1 & -162(x_2 + 0.1) & \cos x_3 \\ -x_2e^{-x_1x_2} & -x_1e^{-x_1x_2} & 20 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(k)} \\ x_2^{(k)} \\ x_3^{(k)} \end{bmatrix} = \begin{bmatrix} x_1^{(k-1)} \\ x_2^{(k-1)} \\ x_3^{(k-1)} \end{bmatrix} - \left(J\left(x_1^{(k-1)}, x_2^{(k-1)}, x_3^{(k-1)}\right)\right)^{-1} \mathbf{F}\left(x_1^{(k-1)}, x_2^{(k-1)}, x_3^{(k-1)}\right)$$

$$J\left(\mathbf{x}^{(k-1)}\right) = \begin{bmatrix} 3 & x_3^{(k-1)} \sin x_2^{(k-1)} x_3^{(k-1)} & x_2^{(k-1)} \sin x_2^{(k-1)} x_3^{(k-1)} \\ 2x_1^{(k-1)} & -162\left(x_2^{(k-1)} + 0.1\right) & \cos x_3^{(k-1)} \\ -x_2^{(k-1)} e^{-x_1^{(k-1)} x_2^{(k-1)}} & -x_1^{(k-1)} e^{-x_1^{(k-1)} x_2^{(k-1)}} & 20 \end{bmatrix}$$

$$\mathbf{F}\left(\mathbf{x}^{(k-1)}\right) = \begin{bmatrix} 3x_1^{(k-1)} - \cos x_2^{(k-1)} x_3^{(k-1)} - \frac{1}{2} \\ \left(x_1^{(k-1)}\right)^2 - 81\left(x_2^{(k-1)} + 0.1\right)^2 + \sin x_3^{(k-1)} + 1.06 \\ e^{-x_1^{(k-1)} x_2^{(k-1)}} + 20x_3^{(k-1)} + \frac{10\pi - 3}{3} \end{bmatrix}$$

$$\mathbf{x}^{(0)} = (0.1, 0.1, -0.1)^t.$$

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$\lVert\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\rVert_\infty$
0	0.10000000	0.10000000	−0.10000000	
1	0.50003702	0.01946686	−0.52152047	0.422
2	0.50004593	0.00158859	−0.52355711	1.79×10^{-2}
3	0.50000034	0.00001244	−0.52359845	1.58×10^{-3}
4	0.50000000	0.00000000	−0.52359877	1.24×10^{-5}
5	0.50000000	0.00000000	−0.52359877	0